Distributing Data for Secure Database Services

Vignesh Ganapathy, Dilys Thomas, Tomas Feder, Hector Garcia-Molina, Rajeev Motwani
Stanford University, TRDDC Pune, Google
E-mail: vignesh@google.com, dilys@cs.stanford.edu, tomas@theory.stanford.edu, hector@cs.stanford.edu, rajeev@cs.stanford.edu

Abstract. The advent of database services has resulted in privacy concerns on the part of the client storing data with third party database service providers. Previous approaches to enabling such a service have been based on data encryption, causing a large overhead in query processing. A distributed architecture for secure database services is proposed as a solution to this problem where data is stored at multiple servers. The distributed architecture provides both privacy as well as fault tolerance to the client. In this paper we provide algorithms for (1) distributing data: our results include hardness of approximation results and hence a heuristic greedy algorithm for the distribution problem (2) partitioning the query at the client to queries for the servers implemented by a bottom up state based algorithm. Finally the results at the servers are integrated to obtain the answer at the client. We provide an experimental validation and performance study of our algorithms.

Keywords. California SB1386, Data Privacy, Distributed Secure Database, SQL Mediation, Normalization, HyperGraph Partitioning, Encryption

1 Introduction

Database service providers are becoming ubiquitous these days. These are companies which have the necessary hardware and software setup (data centers) for storage and retrieval of terabytes of data [8, 14, 25]. As a result of such service providers, parties wanting to store and manage their data may prefer to outsource data to these service providers. The parties who outsource their data will be referred to as clients hereafter. The service providers storing data will be referred to as servers.

There is a growing concern regarding data privacy among clients. Often, client data has sensitive information which they want to prevent from being compromised. Examples of sensitive databases include a payroll database or a medical database. To capture the notions of privacy in a database, privacy constraints are specified by the client on the columns of the sensitive database. We use the notion of privacy constraints as described in [3, 23]. An

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example of a privacy constraint is (age, salary) which states that age and salary columns of a tuple must not be accessible together at the servers. The clients also have a set of queries also known as the workload that need to be executed on a regular basis on their outsourced database.

Most existing solutions for data privacy rely on encrypting data at the server, so that only the client can decrypt it (see for example [16, 17]). Unfortunately, it is hard to run general queries on encrypted data efficiently. If the server cannot execute parts of a query, it sends a fraction of the encrypted database back to the client for further filtering and processing, clearly an expensive proposition.

Instead, Reference [3] suggests using two (multiple) service providers in order to store the data. The advantage of using two servers is that the columns can be split across the two servers to satisfy privacy constraints without encrypting the split columns. Thus, in order to satisfy privacy constraints, columns can either be split across servers or stored encrypted. Thus the goal of any decomposition algorithm is to partition the database to satisfy the following.

- None of the privacy constraints should be violated.
- For a given workload, minimum number of bytes should be transferred between the servers and the client.

We explain both of the above points in detail in the next section. The problem of finding the optimal partition structure for a given set of privacy constraints and query workload can be shown to be intractable. We apply heuristic search techniques based on greedy hill climbing approach to come up with nearly optimal solutions.

The organization of the paper is as follows. After the introduction in Section 1 we provide the system architecture in Section 2. We then provide theoretical intractability results for the schema decomposition problem in Section 3. In Section 4 we provide cost estimation and SQL query answering algorithms, which we use in Section 5 to provide a practical heuristic for the partitioning problem. In Section 6 we provide experimental results on a synthetic and a personal data example and we conclude with related work in Section 7.

2 System Architecture

![Diagram of Distributed Architecture for a Secure Database Service](image)

Figure 1: Distributed Architecture for a Secure Database Service

The general architecture of a distributed secure database service, as illustrated in Figure 1, is described more in [3]. It consists of a trusted client as well as two or more servers that
provide a database service. The servers provide reliable content storage and data management but are not trusted by the client to preserve content privacy. Some relevant terms are described here before going into further details.

- **Data Schema**: This is the schema of the relation the client wishes to store on the server. As a running example, consider a company desiring to store relation \( R \) with the following schema.

\[
R \ (\text{Name}, \text{DoB}, \text{Gender}, \text{ZipCode}, \text{Position}, \text{Salary}, \text{Email}, \text{Telephone})
\]

- **Privacy Constraints**: These are described as a collection of subsets of columns of a relation which should not be accessible together. Single column constraint means that the column should be encrypted. The company may have the following privacy constraints defined:

  \{\text{Telephone}\}, \{\text{Email}\}, \{\text{Name}, \text{Salary}\},
  \{\text{Name}, \text{Position}\}, \{\text{Name}, \text{DoB}\},
  \{\text{DoB}, \text{Gender}, \text{ZipCode}\}, \{\text{Position}, \text{Salary}\}, \{\text{Salary}, \text{DoB}\}

- **Workload**: A workload \( W \) is a set of queries that will be executed on a regular basis on the client’s data.

- **Tuple ID (TID)**: Each tuple of the relation is assigned a unique tuple ID. The TID is used to merge data from multiple servers when executing a query on the data. The use of TID will become more explicit in the query plans described next.

- **Partitions**: There are two servers to store the client database. The schema and data are partitioned vertically and stored at the two servers. A partition of the schema can be described by three sets \( R_1 \) (attributes of \( R \) stored on Server 1), \( R_2 \) (attributes of \( R \) stored on Server 2) and \( E \) (set of encrypted attributes stored on both servers). It is important to note that \( (R_1 \cup R_2 \cup E) = R \) and it is not necessarily the case that \( R_1 \cap R_2 = \emptyset \). We denote a decomposition of \( R \) as \( D(R) \). An example decomposition \( D(R) \) of \( R \) is given here.

  Partition 1 \( \{R_1\} \): \( \{\text{TID}, \text{Name}, \text{Email}, \text{Telephone}, \text{Gender}, \text{Salary}\} \)

  Partition 2 \( \{R_2\} \): \( \{\text{TID}, \text{Position}, \text{DoB}, \text{Email}, \text{Telephone}, \text{ZipCode}\} \)

  Encrypted Attributes \( E \): \{\text{Email}, \text{Telephone}\}

- **Query Execution Plans in Distributed Environment**: When data is fragmented across multiple servers, there are two plan types used frequently to execute queries on data stored on these servers.

  - **Centralized Plans**: On execution of a query, data from each server is transmitted to the client and all further processing is done at the client side. In some cases, multiple requests can go to each server but data from one server is never directly sent over to the other servers.

  - **Semi Join Plans**: As an alternative to centralized plans, it may be more efficient to consider semi join plans. Here, TIDs are passed from one server to the other to reduce the amount of traffic flow to the client.

- **Encryption Details**: Encryption of columns can either be deterministic or non-deterministic. A deterministic encryption is one which encrypts a column value \( k \) to the same value \( E(k) \) every time. Thus, it allows equality conditions on encrypted columns to be executed on the server. Our implementation assumes encryption on columns to be deterministic.

- **Column Replication**: When columns of a relation are encrypted, then they can be placed in any of the two servers since they will satisfy all privacy constraints. It is beneficial to store the encrypted columns on both servers to make query processing more...
efficient. Non encrypted columns can also be duplicated as long as privacy constraints are also satisfied. Replication will result in lesser network traffic most of the time. To prevent associations semantic security with randomized encryption can be used at the price of higher query processing cost.

- **Cost Overhead:** We model the cost as the number of bytes transmitted on the network assuming that this supersedes the I/O cost on the servers and processing cost on the client. Cost overhead is the parameter used to determine the best possible partitioning of a relation. It measures the number of excess bytes transferred from the server to the client due to the partition.

\[
\text{Cost Overhead}(D(R)) = X - Y,
\]

where \(X = \text{Bytes transmitted when executing workload } W \text{ on a decomposition } D(R) \text{ of } R \text{ at two servers},\)

\(Y = \text{Bytes transmitted when executing workload } W \text{ on relation } R \text{ at one server with no fragmentation}.\)

The problem can now formally be defined as follows.

We are given: (1) A data schema \(R\); (2) A set of privacy constraints \(P\) over the columns of the schema \(R\); (3) A workload \(W\) defined as a set of queries over \(R\). We have to come up with the best possible decomposition \(D(R)\) of the columns of \(R\) into \(R_1, R_2\) and \(E\) such that:

(1) All privacy constraints in \(P\) are satisfied. These can either be satisfied by encrypting one or more attributes in the constraint or have at least one column of the constraint at each of the servers. Encrypting columns has its disadvantages as discussed before so we give priority to splitting columns as a way to satisfy privacy constraints. (2) The cost overhead of \(D(R)\) for the workload \(W\) should be the minimum possible over all decompositions of \(R\) which satisfy \(P\). Space is not considered as a constraint and columns of relations are replicated at both servers as long as they satisfy privacy constraints.

### 3 Intractability of Schema Decomposition

In this section, we provide hardness of approximation results for the schema decomposition problem. These results are not essential to understand the rest of the paper. They provide a formal reasoning why simplified versions of the schema decomposition problems are hard to approximate.

A standard framework to capture the costs of different decompositions, for a given workload \(W\), is the notion of the affinity matrix [24] \(M\), which we adopt and generalize as follows:

- The entry \(M_{ij}\) represents the performance “cost” of placing the unencrypted attributes \(i\) and \(j\) in different fragments.
- The entry \(M_{ii}\) represents the “cost” of encrypting attribute \(i\) across both fragments.

We assume that the cost of a decomposition may be expressed simply by a linear combination of entries in the affinity matrix. Let \(R = \{A_1, A_2, \ldots, A_n\}\) represent the original set of \(n\) attributes, and consider a decomposition of \(D(R) = (R_1, R_2, E)\), where \(R_1\) is at Server 1, \(R_2\) at Server 2 and \(E\) the set of encoded attributes. Then, we assume that the cost of this decomposition \(C(D)\) using [3] is \(\sum_{i\in(R_1-E),j\in(R_2-E)} M_{ij} + \sum_{i\in E} M_{ii}\). (For simplicity, we do not consider replicating any unencoded attribute, other than the tupleID, at both servers. The hardness results hold nonetheless.)

In other words, we add up all matrix entries corresponding to pairs of attributes that are separated by fragmentation, as well as diagonal entries corresponding to encoded at-
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Given a set of privacy constraints \( \mathcal{P} \subseteq 2^R \) and an affinity matrix \( M \), find a decomposition \( D(R) = \langle R_1, R_2, E \rangle \) such that

(a) \( D \) obeys all privacy constraints in \( \mathcal{P} \), and
(b) \( \sum_{i,j \in (R_1 - E), j \in (R_2 - E)} M_{ij} + \sum_{i \in E} M_{ii} \) is minimized.

We model the above problem with a graph theoretic abstraction. Each column of the relation is modeled as a vertex of the graph \( G(V,E) \), whose edge weights are \( M_{ij} \) and vertex weights are \( M_{ii} \). We are also given a collection of subsets \( \mathcal{P} \subseteq 2^R \), say \( S_1, \ldots, S_t \) which model the privacy constraints. Given this graph \( G(V,E) \) with both vertex and edge non-negative weights, our goal is to partition the vertex set \( V \) into three subsets - \( E \) (the encrypted attributes), \( R_1 \) (the attributes at Server 1) and \( R_2 \) (the attributes at Server 2). The cost of such a partition is the total vertex weight in \( E \), plus the edge weight of the cut edges from \( R_1 \) to \( R_2 \). However, the constraint is that none of the subsets \( S_1, \ldots, S_t \) can be fully contained inside either \( R_1 \) or \( R_2 \). The closely related minimum graph homomorphism problem was studied in [5].

3.1 Minimum Cut when there are Few Sets \( S_i \)

There is an algorithm that solves the general problem, but this algorithm is efficient only in special cases, as follows.

**Theorem 1.** The general problem can be solved exactly in time polynomial in \( \prod_i |S_i| = n^{O(t)} \) by a minimum cut algorithm, so the general problem is polynomial if the \( S_i \) consist of a constant number of arbitrary sets, a logarithmic number of constant size sets, \( O(\log n / \log \log n) \) sets of polylogarithmic size, and \( (\log n)^{\epsilon} \) sets of size \( e^{(\log n)^{1-\epsilon}} \) for a constant number of distinct \( 0 < \epsilon < 1 \).

**Proof.** One may try in all possible ways to select for each \( S_i \) either one element to go to \( E \) or two elements to go to \( R_1, R_2 \) respectively. Merge together the identified elements of each of \( E, R_1, R_2 \) next, remove the identified elements in \( E \), and now we are looking for a min cut between the identified \( R_1, R_2 \) given by vertices that go to \( E \) and edges that join \( R_1, R_2 \). In fact we may turn the edges into vertices by putting a middle vertex of the appropriate weight on each edge, so we are looking for a min-vertex cut, which is polynomial. Thus the complexity is polynomial in \( \prod_i |S_i| = n^{O(t)} \) because of the initial number of possible choices.

3.2 Minimum Hitting Set when Solutions do not Use \( R_2 \)

When edges have infinite weight, no edge may join \( R_1 \) and \( R_2 \) in a solution.

In the hitting set problem we are asked to select a set \( E \) of minimum weight that intersects all the sets in a collection of sets \( S_i \).

**Theorem 2.** For instances whose edges form a complete graph with edges of infinite weight the problem is equivalent to hitting set, and thus has \( \Theta(\log n) \) easiness and hardness of approximation.

**Proof.** We may not cross the cut \( R_1, R_2 \) as this would give infinite cost. We may thus assume that only \( E, R_1 \) will be used. Each set \( S_i \) must then have at least one element in \( E \), so a solution is valid only if \( E \) hits all the sets \( S_i \), and the cost is the sum of the vertex weights in the hitting set \( E \).
3.3 The case $|S_i| = 2$ and Minimum Edge Deletion Bipartition

When vertices have infinite weight, no vertex may go to $E$.

In the minimum edge deletion bipartition problem we are given a graph and the aim is to select a set of edges of minimum weight to remove so that the resulting subgraph after deletion is bipartite. This problem is constant factor hard to approximate even when the optimum is proportional to the number of edges, as shown by Hastad [19], can be approximated within a factor of $O(\log n)$ as shown by Garg, Vazirani, and Yannakakis [11], and within an improved factor of $O(\sqrt{\log n})$ as shown by Agarwal et al. [2].

The next three results compare the problem having $|S_i| = 2$ to minimum edge deletion bipartition.

**Theorem 3.** If all vertex weights are infinite (so that $E$ may not be used), the sets $S_i$ are disjoint and have $|S_i| = 2$, and all edge weights are 1, then the problem encodes the minimum edge deletion bipartition problem and is thus constant factor hard to approximate even when the optimum has value proportional to the number of edges.

**Proof.** Encode each vertex $v$ of $G$ as $S_v = \{a_v, b_v\}$, and encode each edge $vw$ of $G$ as the two edges joining $S_v, S_w$ given by $a_v, b_w, b_v, a_w$. The side of the bipartition for $v$ depends on whether $a_v$ or $b_v$ goes to $R_1$, and an edge $vw$ is removed if $v, w$ go to the same side, in which case we pay for both $a_v b_w, b_v a_w$ across the cut $R_1, R_2$. Thus the problem is equivalent to the minimum edge deletion bipartition. \qed

**Theorem 4.** If all vertex weights are infinite (so that $E$ may not be used), the sets $S_i$ have $|S_i| = 2$, then the problem may be approximated in polynomial time by a minimum edge deletion bipartition instance giving an $O(\sqrt{\log n})$ approximation.

**Proof.** If two sets $S_i$ share an element, say $S_i = \{a, b\}$ and $S_j = \{a, c\}$, then we may merge $b$ and $c$. We may thus assume the $S_i$ are disjoint. Now represent $S_v = \{a_v, b_v\}$ by a vertex $v$, and if $S_v, S_w$ are joined by edges we must pay at least one for these edges, unless these edges are (1) contained in $a_v b_w, b_v a_w$ or (2) contained in $a_v a_w, b_v b_w$. In case (1) we join $vw$ by an edge, and in case (2) we introduce a new vertex $w$ and form a path $vwv$ of length 2. This encodes the problem as a minimum edge deletion bipartition problem up to a constant factor, so an $O(\sqrt{\log n})$ approximation exists. The problem with edge weights is similarly solved by subtracting weights joining $S_i$ and $S_j$ until we fall in cases (1) or (2) above. \qed

**Theorem 5.** If all vertex weights are 1, there are no edges, and the sets $S_i$ have $|S_i| = 2$, then the problem encodes minimum edge deletion bipartition and is thus hard to approximate within some constant even for instances that have optimum proportional to the number of vertices.

**Proof.** If we consider the sets $S_i$ as edges, this is the problem of removing the least number of vertices to make the graph bipartite. We know hardness for removing edges to make the graph bipartite. To translate to vertices, separate each vertex of degree $d$ into $d$ vertices, one for each adjacency, and connect these $d$ vertices with a constant degree expander graph, where each edge of the expander graph is replaced with constant number of parallel paths of length two. Thus if less than half the vertices of the same expander graph get removed, this corresponds to removing a proportional number of edges. \qed

We now approximate the general problem with sets $S_i$ having $|S_i| = 2$. The performance is similar to the minimum vertex deletion problem.
Theorem 6. The general problem with sets $S_i$ having $|S_i| = 2$ can be solved with an approximation factor of $O(\sqrt{n})$ by directed multicut.

Proof. Represent each vertex $v$ of weight $x$ by four vertices $a_v, b_v, c_v, d_v$ joined by arcs $a_v b_v, c_v d_v$ of capacity $x$. Represent each set $S_i = \{x, y\}$ by arcs $b_v c_v, d_v a_v, b_v c_v, d_v a_v$ of infinite capacity. Represent each edge $uv$ of weight $y$ by arcs $b_v a_v, d_v c_v, b_v a_v, d_v c_v$ of capacity $y$. Finally look for a multicut that separates the sources $a_v$ from the corresponding sinks $d_v$. Removing an arc for a vertex $x$ corresponds to assigning $x$ to $E$, and after removing arcs corresponding to $uv$, the vertices form by reachability two components $R_1$ for arcs $a_v, b_v$ and $R_2$ for arcs $c_v, d_v$. The multicommodity flow result of Gupta [15] for directed multicut gives the $O(\sqrt{n})$ bound.

\[ \square \]

3.4 The case $|S_i| = 3$ and Intractability

The problem with $|S_i| = 3$ becomes much harder to approximate, compared to the $O(\sqrt{n})$ factor for $|S_i| = 2$.

Theorem 7. If all vertex weights are 1, there are no edges, and the sets $S_i$ have $|S_i| = 3$, then the problem encodes not-all-equal 3-satisfiability and it is thus hard to distinguish instances of zero cost from instances of cost proportional to the number of vertices.

Proof. If there are are no edges and the sets $S_i$ have size $|S_i| = 3$, then the problem encoded is not-all-equal 3-satisfiability by corresponding sets $R_1$ and $R_2$ to values 0 and 1 respectively. Even satisfiable instances of not-all-equal 3-satisfiability have a constant factor hardness on the number of variables participating in unsatisfied clauses by a solution. We conclude that a constant fraction of such variables must be assigned to $E$ even if a zero cost solution exists. The hardness of approximation of non-all-equal satisfiability for number of variables instead of clauses is obtained by making multiple copies of the same variable for multiple clauses, and joining these with a constant degree expander graph. Each edge $xy$ of the expander graph represents a path of length two $xzy$, where $xz$ and $zy$ represent $x \neq z$ and $z \neq y$ over $\{0, 1\}$ respectively. We represent $x \neq y$ with clauses $\{x, y, z\}, \{x, u, y\}, \{x, v, y\}, \{t, u, v\}$. The result thus follows from the result for not-all-equal satisfiability of Håstad [19].

\[ \square \]

We examine the tractability when the sets $S_i$ are disjoint.

Theorem 8. If all vertex weights are infinite (so that $E$ may not be used), the sets $S_i$ are disjoint and have $|S_i| = 3$, and all edge weights are 1, then the problem encodes not-all-equal 3-satisfiability and it is thus hard to distinguish instances of zero cost from instances of cost proportional to the number of edges.

Proof. A clause of not-all-equal satisfiability may be viewed as a set $S_i = \{x_i, y_i, z_i\}$. We assume these $S_i$ are disjoint and join copies of variables in different clauses by a clique. If the not-all-equal 3-satisfiability problem has a solution, a solution of zero cost exists for our problem. The number of clauses satisfied in a not-all-equal satisfiability problem is constant factor hard to approximate even on instances that are satisfiable. Therefore in a solution to our problem a constant fraction of the sides chosen for the elements of the $S_i$ would have to be changed between $R_1$ and $R_2$ to obtain a consistent solution to not-all-equal 3-satisfiability that fails a constant fraction of the clauses. If we replace each clique by a constant degree expander graph, then each of the elements of $S_i$ that would be changed between $R_1$ and $R_2$ pays a constant, as at most half of the elements of the expander graph
for a clique are changed. Thus we pay cost proportional to the number of edges when the optimal cost is zero by hardness of approximation of not-all-equal 3-satisfiability shown by Hastad [19].

**Theorem 9.** The problem with vertices of infinite weight and edges of weight 1, sets $S$, with $|S_i| = 3$ forming a partition with no edges within an $S_i$, the graph $H_{1,2,3,1',2',3'}$ with no edges joining $S = \{1, 2, 3\}$ and $S' = \{1', 2', 3'\}$ allowed, can be classified as follows:

1. If only additional $H$ from $K_0$ are allowed, the problem is constant factor approximable;
2. If only additional $H$ from $K_0$ and $K_1$ are allowed, the problem is $O(\log n)$ approximable; furthermore as long as some graph from $K_1$ is allowed, the problem is no easier to approximate than minimum edge deletion bipartition, up to constant factors.
3. If only additional $H$ from $K_0$, $K_1$ and $K_2$ are allowed, the problem is $O(\log n)$ approximable;
4. If some additional $H$ from $K_3$ is allowed it is hard to distinguish instances with cost zero from instances with cost proportional to the number of edges.


We next prove (1). Any occurrences of $H_{1231',2',3,3',3'}$, $H_{1231',2',3,3',3',3'}$, $H_{1231',2',3,3',3',3'}$ must pay in a solution, so we may remove these and pay cost proportional to the number of these. The graphs $H_{121',2',3,3',3'}$, $H_{121',2',3,3',3'}$ are equivalent up to constant factors, so we consider just $H_{121',2',3,3',3'}$. If 12 are combined in $H_{121',2',3,3',3'}$ and 13 are combined in $H_{121',2',3,3',3'}$, then at least one of these two graphs must pay, so for each set $S = \{1, 2, 3\}$ we may consider the number $a_{12}$ of graphs combining 12, the number $a_{13}$ of graphs combining 13, and the number $a_{23}$ of graphs combining 23, and remove the least two of $a_{12}$, $a_{13}$, $a_{23}$ number of graphs (say remove the graphs for combinations 13 and 23 and keep the graphs for combinations 12). This incurs another constant factor of the optimum. Finally every $S$ has only one combination, so we may assign 12 to $R_1$ and 3 to $R_2$ at zero cost. This proves (1).

We next prove (2). Define $\hat{H}_{11',2',2',3,3',3'}$ by $H_{11',2',2',3,3',3'}$, $H_{11',2',2',3,3',3'}$, $H_{11',2',2',3,3',3'}$. Thus $\hat{H}_{11',2',2',3,3',3'}$ is $H_{11',2',2',3,3',3'}$ plus the condition that $11'$ and $22'$ go to $R_1$ and $R_2$ respectively or to $R_2$ and $R_1$ respectively. We may remove occurrences of the first three graphs $H$ in $K_0$ by paying cost proportional to the number of such $H$ as before. The last two $H$ in $K_0$ and the two $H$ in $K_1$ can be simulated by $\hat{H} = H_{11',2',2',3,3',3'}$, as they are superpositions of several copies of $H$ under various permutations of the elements of $S$ and $S'$, and superpositions may be avoided by concatenating two copies of $H$ to obtain $\hat{H}$ again. We may thus suppose that $\hat{H}$ is the only graph that occurs. Suppose the role of 3 in $\hat{H}$ for $S = \{1, 2, 3\}$ is played in different groups by 1, 2, 3, so that we have $\hat{H}_{11',2',2',3,3',3'}$, $\hat{H}_{11',2',2',3,3',3'}$, $\hat{H}_{11',2',2',3,3',3'}$. Then one of these three must pay, so we may remove for $S$ one group that occurs in the least number of such graphs (say keep the first to with the role of 3 played by 2 or 3 and remove the ones where the role of 3 is played by 1). The cost paid is proportional to the number of such graphs removed, incurring a constant factor approximation. We may finally assume that only $\hat{H}_{12',21',3,3',3'}$, $\hat{H}_{13',23',2,2',2'}$ occur. If $\hat{H}_{12',21',3,3',3'}$ occurs, it can be simulated as $\hat{H}_{12',21',3,3',3'}$, $\hat{H}_{12',21',3,3',3'}$. Thus we may say that 1 crosses in such occurrences with either 2 or 3, and so the instance can be solved at zero cost if and only if the graph whose vertices are the $S_i$ and the edges are the $H_{12',21',3,3'}$ joining them is bipartite. The problem thus reduces to the minimum edge deletion bipartition problem and is solvable in time $O(\sqrt{\log n})$. It can be shown that as long as some graph in $K_1$ is allowed, the problem is no easier than minimum edge deletion bipartition, up to constant factors. This proves (2).
We finally prove (3). Suppose first only $H_{11',22',33'}$ occurs. This graph permutes 123 into $1'2'3'$ in some way. We may compose such permutations and come back to $S = \{1, 2, 3\}$. If 123 comes back as 231, then the instance has no solution of zero cost. If this never happens, say 123 only comes back as 213, then we may map 1, 2 to $R_1$ and 3 to $R_2$, obtaining a solution of zero cost. We may thus represent each such $S$ by six vertices corresponding to the six permutations 123, 132, 213, 231, 312, 321, and match the six permutations for $S = \{1, 2, 3\}$ to the six permutations for $S' = \{1', 2', 3'\}$. On this graph with six vertices for each $S$, we may look for a multicut separating each pair 123, 231 for each $S$. This can be done by the algorithm of Garg, Vazirani, and Yannakakis [11] with an $O(\log n)$ approximation. This proves the case of $H_{11',22',33'}$ alone.

We complete the proof of (3). The first three sets $H$ of $K_0$ pay as from before and are removed at a constant factor approximation. The last two sets $H$ of $K_0$ and the two sets $H$ of $K_1$ can be simulated by $H_{11',22',3,3'}$ defined as before. The part of the problem involving only $H_{11',22',3,3'}$ can be solved with an $O(\log n)$ approximation, by removing the corresponding multicut. Now if a set $S$ is connected directly to $d$ other $S_i$, then create $d$ copies of $S$, one for each $S_i$, and join the copies of $S$ with a constant degree expander graph involving edges between this copies having $H_{11',22',3,3'}$. Finally for each occurrence of $H_{11',22',3,3'}$ at $S = \{1, 2, 3\}$ (there is now at most one such occurrence at $S$) ask for a multicut separating $1, 2$ for such $S$, as $1, 2$ must go to $R_1, R_2$ respectively or $R_2, R_1$ respectively. This is again done with an $O(\log n)$ approximation by the algorithm of Garg, Vazirani, and Yannakakis [11]. This completes the proof of (3).

We finally note that for dense instances with $n$ vertices, $m$ edges and sets $S_i$ of constant size, we may apply the techniques of Alon et al. [7] to solve the problem within an additive $\epsilon \cdot m$ in time $2^{O(n^2/(\epsilon^2 m))}O(n^2)$ for $m = |E(G)|$.

### 4 Cost Estimation

Since algorithms with theoretical guarantees are hard to obtain for our data partition problem (Section 3), we develop instead a heuristic search strategy that finds good, although not optimal, solutions.

Figure 2 illustrates our approach to finding good partitions. At the core is a hill climbing module that tries to improve on an existing partition. This module starts with an initial, simple partition (Section 5) that satisfies the privacy constraints, and then makes local changes to the partition that still satisfy the constraints. To decide if a partition is better than the current one, the hill climbing module must compare their costs. For this comparison, it uses two modules: (1) the translation engine that given a workload query and a partition, determines the execution plan (what sub-queries are sent to the servers); and (2) the query cost estimator that estimates the cost of a given plan.

#### 4.1 Query cost estimator

We limit the type of queries as defined by the grammar in Figure 3. The grammar specifies the valid predicates that we support as part of the WHERE clause of a SQL query.

In order to perform cost estimation, we collect and maintain statistics of the data. For a given relation $R_r$ we maintain the following information.

$T(R_r)$: Number of tuples in $R_r$

$S(R_r, a)$: Size in bytes of attribute $a$ in $R_r$
Figure 2: Components of the Partitioning Algorithm

\[ S ::= (\Sigma, a, c) \]
\[ \Sigma \in \mathbb{P} \rightarrow \text{Predicate} \]
\[ \text{Predicate} \ni F ::= (F_1 \land F_2) \mid (F_1 \lor F_2) \mid a = c \mid a \leq c \mid c \leq a \mid c_1 \leq a \mid c_2 \mid a_1 = a_2 \]
\[ a \in \mathbb{A} = \{ T.x, T.y, ... \} \]
\[ c \in \mathbb{Z} = \{ ..., -1, 0, 1, ... \} \]

Figure 3: Syntax of the Boolean Predicate

\( V(a) \): Number of distinct values of a in \( R \)

Let \( F \) be a boolean predicate which is given by the grammar in Figure 3. We use \( \rho(F) \) to denote the selectivity of the formula \( F \). \( \rho(F) \) is computed recursively using the semantics given in Figure 4.

The attributes in the SELECT clause of the query decide the size in bytes of each result tuple. Query cost, \( QC(q) \) represents the size estimation for query \( q \) and \( SL(q) \) is the set of attributes in the SELECT clause for \( q \). The cost estimate for a partition is computed as the sum of the cost estimate of the two queries.

\[ QC(q) = (\sum_{i \in SL(q)} S(R, i)) \ast T(R) \ast \rho(F) \]

4.2 Translation and Execution Engine

The translation engine is the system component which generates SQL queries for the decomposition \( D(R) \) of \( R \), given a SQL query on \( R \). The partitioned queries generated by the engine can now be fed to the query estimator discussed in the previous section to obtain cost estimates for each query. The type of plan used is an important factor which decides the form of the resulting queries. For the purposes of this paper, we generate queries for centralized plans.

This problem of deciding which server to use to access data is better known are data localization in distributed databases theory as discussed in [24]. Replication and encryption
\[
\rho(F) = \begin{cases} 
\frac{1}{V(a)} & \text{if } a = c \\
\frac{c - \min(a) + 1}{\max(a) - \min(a) + 1} & \text{if } a \leq c \\
\frac{c - \min(a) + 1}{\max(a) - \min(a) + 1} & \text{if } c \leq a \\
\frac{\max(V(a_1), V(a_2))}{1 - (1 - \rho(F_1))(1 - \rho(F_2))} & \text{if } F = F_1 \lor F_2 \\
\rho(F_1) \times \rho(F_2) & \text{if } F = F_1 \land F_2 
\end{cases}
\]

Figure 4: Semantics of the Boolean Predicate

add more complexity to the localization process. For example, if an attribute is available at both servers, one decision to make is which copy must be accessed. Range queries on encrypted attributes will require the entire column to be transmitted to the client for decryption before determining the results of the query. Decisions like which copy to access cannot be determined locally and individually for each condition clause.

We propose a technique which computes the where clauses in the decomposed queries in two steps. We define two types of state values, \(W\) and \(S\) each of which provide information as to which servers to access for the query execution. We process the WHERE clause to get \(W\) and then process the SELECT clause to get \(S\). In the final step, we use both these values and the corresponding select and condition list to determine the decomposed queries. We use the schema \(R\) and decomposition \(D(R)\) defined in section 2. Most of the steps that follow are part of query localization which is to decide which part of the query is processed by which server.

4.2.1 WHERE clause processing

The basic units of the WHERE clause are conditional clauses where operators could be \(>\), \(<\) or \(=\) as per the grammar defined above. Each of these basic units are combined using the AND or the OR operator. The entire clause itself can be effectively represented by a parse tree. Such a parse tree has operators as non-leaf nodes and operands as leaf nodes.

**Bottom Up State Evaluation:**
Bottom up evaluation of the parse tree starts at the leaf nodes. Each node transmits to its parent, its state information. The parent operator (always a binary operator) will combine the states of its left and right subtrees to generate a new state for itself.

**State Definitions:**
Each node in the tree is assigned a state value. Let \(W\) be the state value of the root of the parse tree. The semantics of the state value are as follows.

0: condition clause cannot be pushed to either servers;
1: condition clause can be pushed to Server 1;
2: condition clause can be pushed to Server 2;
3: condition clause can be pushed to both servers;
4: condition clause can be pushed to either servers.

As we proceed to determine the state values for all nodes, we need to consider nodes with state value 0 as a special case. All child attributes for a node with state value 0 are added to the select list of the query.

There are three cases to consider for a non-leaf node.

**Case 1:** The parent node is one of the operators \(>, <, =\).
Table 1: AND operator state table

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
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</tbody>
</table>

Table 2: OR operator state table

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
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<td>4</td>
</tr>
</tbody>
</table>

For such a parent node, the child nodes are attributes of relations or constant values. If the condition is an attribute-value clause, the state of the parent is the state value of the attribute. The state value of the attribute in turn is determined by the location of that attribute. If the condition is an attribute-attribute clause (a1 = a2 or a1 < a2 etc), the state of the parent is:

0 if the state values of the attributes are (1,2) or if they are on the same server but one of the attributes is encrypted.; 1 if the state values of the attributes are (1,1) or (1,4) ; 2 if the state values of the attributes are (2,2) or (2,4); 4 if the state values of the attributes are (4,4)

Case 2: The parent node is the AND operator

Table 1 represents the state determination of an AND parent node given the state values of its two children. The row in the table is the state of the first child, the column is the state of the second child and the table entry is the state of the parent. For example, (4,3) implies a state 3 for the parent using the AND table.

Case 3: The parent node is the OR operator. Table 2 represents the state determination of an OR parent node given the state values of its two children.

4.2.2 SELECT clause processing

The select part of each query can be a set of attributes of the updated relations in the schema. The select clause processing generates a state S and two sets of attributes A1 and A2. S represents the following cases.

1: Requires access to Server 1 only; 2: Requires access to Server 2 only; 3: Requires access to both servers; 4: Requires access to any of the two servers.

A1 and A2 are the attributes in the select clause that are present on Server 1 and Server 2 respectively. If the attribute is present on both servers, it will be contained in A1 and A2.

4.2.3 Final Query Decomposition step

The final step is to generate the two queries Query 1 (to be sent to Server 1) and Query 2 (to be sent to Server 2). We use the state value of the root node obtained from the WHERE
clause processing \( W \) and the SELECT clause state value \( S \). There are five cases depending on the state value of the root node.

\( W = 0 \):
There are no where clauses in Query 1 and Query 2 since none of the conditions can be pushed to the servers.

\( W = 1 \) or \( 2 \):
For state value 1, Query 2 does not contain any where clause. Similarly, for state value 2, Query 1 does not have any where clause.

\( W = 3 \) - Top Down processing of Clauses:
We perform a top-down processing of the tree. We start at the root operator and proceed downwards as long as we encounter state 3. We thus stop when we are sure whether to include the clause as part of Query 1 (state value 1), Query 2 (state value 2), Query 1/Query 2 (state value 4) or not to include it at all (state value 0).

\( W = 4 \):
The where clauses can be pushed completely to any one of the two servers.

Let us build on the running example from Section 2 to see how all this fits in to solve the problem as a whole. Consider a slightly more complicated client query.

```
SELECT Name, DoB, Salary FROM R
WHERE (Name = 'Tom' AND Position='Staff') AND (Zipcode = '94305' OR Salary > 60000)
```

Let the predicates in the query be assigned \( P_1 \) (Name = 'Tom'), \( P_2 \) (Position='Staff'), \( P_3 \) (Zipcode='94305') and \( P_4 \) (Salary > 60000). The parse tree and the corresponding state values are shown in Figure 5.

```
Figure 5: State Computation for the Predicate
```

Query1: SELECT TID, name, salary
FROM R1 WHERE Name='Tom'
Query2: SELECT TID, dob, zipcode
FROM R2 WHERE Position='Staff'
The query plan is also shown here in Figure 6 detailing out the steps that need to be performed for executing this query.

In the plan, \( T, N, S \) and \( D \) stand for TID, Name, Salary and DoB attributes of schema \( R \) respectively. At the client side, results of Query1 and Query2 are joined on attribute TID.
The predicates P3 and P4 are then applied to the results followed by a projection on the select attributes.

\[
\text{T6} \leftarrow \text{SELECT}[\text{P3 OR P4}](\text{T5})
\]

\[
\text{T5} \leftarrow (\text{T2 JOIN}[\text{T2.T=T4.T}]*\text{T4})
\]

\[
\text{T2} \leftarrow \text{PROJECT}[\text{T,N,S}](\text{T1})
\]

\[
\text{T4} \leftarrow \text{PROJECT}[\text{T,D,Z}](\text{T2})
\]

\[
\text{T1} \leftarrow \text{SELECT}[\text{P1}](\text{R1})
\]

\[
\text{T3} \leftarrow \text{SELECT}[\text{P2}](\text{R2})
\]

\[
\text{T6} \leftarrow \text{PROJECT}[\text{N,D,S}](\text{T6})
\]

Figure 6: Distributed Query Plan

5 Partitioning Algorithms

Hill-climbing is a heuristic in which one searches for an optimum combination of a set of variables by varying each variable one at a time as long as the objective value increases. The algorithm terminates when no local step decreases the cost. The algorithm converges to a local minima.

An initial fragmentation of the database is considered which satisfies all the privacy constraints.

Initial Guess: The initial set of encrypted attributes is obtained using the weighted set cover given in Algorithm 1.

Algorithm 1 Initial Attributes For Encryption

\begin{center}
\begin{tabular}{l}
\text{Encrypted = \{} Attributes in singleton privacy constraints } \\
\text{CanConsider = \{} Remaining attributes occurring in atleast one privacy constraint } \\
\text{RemConstraints = \{} All non singleton privacy constraints } \\
\end{tabular}
\end{center}

\begin{center}
\textbf{while RemConstraints is not Empty} do
\end{center}

\begin{center}
(Re)Assign weights to all elements in CanConsider based on workload encryption cost
\end{center}

\begin{center}
Pick attribute e, from CanConsider, with minimum weight / (divided by) number of constraints it appears in.
\end{center}

\begin{center}
Add e to Encrypted
\end{center}

\begin{center}
Delete all privacy constraints in RemConstraints that contain e
\end{center}

\begin{center}
Delete e from CanConsider
\end{center}

\begin{center}
end while
\end{center}

The result is the attribute set Encrypted.
Hill Climbing Step: The hill climbing algorithm is applied after the initial encryption as shown in Algorithm 2.

**Algorithm 2 Local Search Heuristic to Reduce Cost**

```latex
\begin{algorithm}
\textbf{while} Privacy Constraints are Satisfied and Cost Reduces \textbf{do}
  \begin{itemize}
    \item Try decrypting an encrypted column and placing it at Server 1.
    \item Try decrypting an encrypted column and placing it at Server 2.
    \item Try decrypting an encrypted column and placing it at both servers
    \item Try encrypting an decrypted column and placing it at both servers.
  \end{itemize}
\textbf{end while}
\end{algorithm}
```

From the steps in the local search heuristic, the one which satisfies privacy constraints and results in minimum network traffic is considered as the new fragmentation and the process repeats. The iterations are performed as long as we get a decomposition at each step which improves over the existing decomposition using the cost metric discussed before.

In order to compare the results produced by our hill climbing strategy with the optimal solution, we also implemented a brute force algorithm. This algorithm considers all possible partitions that satisfy the privacy constraints and selects the one with minimal cost. Note that for a relation with $n$ columns there are $4^n$ possible fragmentations possible and very few of them will satisfy all the privacy constraints. (The “4” arises because there are 4 choices for each attribute: store decrypted at server 1 or 2 or both, or store encrypted at both servers.)

6 Experimental Results

6.1 Details of Experimental Setup:

We execute our code on a single relation $R$ in all experiments. The number of attributes in $R$ was varied from 1 to 30. The number of tuples in $R$ was between 1000 and 10,000. Note that for our experiments we do not actually need the data, only the statistics that describe the data. Thus, the results we obtain for this setting are applicable to relations of a larger (or smaller) size.

As we vary the number of attributes, we generate privacy constraints over it randomly with the following properties. We generate as many privacy constraints as the number of attributes in the relation. Privacy constraints vary in size from one to the number of attributes in the relation. The attributes that are part of each constraint are selected at random from the available attributes without replacement.

Another important parameter is the workload. We generated 25 workloads containing a fixed number of queries (5 in our case) for different number of attributes of the relations. So, for a relation with fixed number of attributes, we generate about 125 queries ($25^5$) divided into twenty five workloads. For each query, the parts that were varied were the attribute set in the SELECT clauses and the conditions in the WHERE clause. We selected a subset of SQL which mapped to our grammar defined in Section 4. Each condition clause $C$ was of the form $(xOPy)$ where $OP$ was chosen at random to be $>$, $<$ or $\leq$. $x$’s were chosen from the columns of $R$ while $y$ was chosen from the domain of $x$ after choosing $x$. The $C$’s themselves were combined by choosing one of OR, AND. The other parameters that were randomly chosen were the number of condition clauses, the number of attributes in the SELECT clause and the actual attributes in the SELECT clause itself.
6.2 Synthetic Data Generation

We conduct experiments to demonstrate how well hill climbing compares with brute force. For each algorithm, we vary the number of attributes $N$ from 1 to 6. For the hill climbing experiment we use ten different workloads for each $N$ and each workload was composed of five queries. While we can obtain results for hill climbing for larger $N$, the brute force approach starts to get intractable. Hill climbing starts to move away from the optimal solution with increasing $N$. For 3 attributes, hill climbing is around 20 percent away from brute force but for 6 attributes, this percentage goes up to around 140.

Next, we study the behavior of hill climbing in terms of the number of iterations it takes to converge. We vary the number of attributes $N$ for the relation from 1 to 30 for these experiments. The number of workloads for each $N$ is 10 and similar to the previous experiment, we have 5 queries per workload.

Figure 7 shows the number of workloads for which the hill climbing converged. We note that the number of workloads requiring more than 10 iterations is less than 2 percent of the workloads.

We now show the improvements achieved by the hill climbing over the initial partition. The improvement percentage is defined as:

$\text{Perc. Improvement} = \frac{(C_{fp} - C_{ip})}{C_{ip}} \times 100$

where $C_{fp} = \text{Cost estimate for the final partition of } R \text{ returned by Hill Climbing for a given workload } W \text{ and set of privacy constraints } P$; and $C_{ip} = \text{Cost estimate of the initial partition given as input to the hill climbing algorithm for the same workload } W \text{ and set of privacy constraints } P$.

Figure 8 illustrates that the number of workloads with greater than 20 percent difference from the initial solution keeps decreasing with increasing percentages. Despite this fact, more than 50 percent of the workloads have a percentage improvement of over 25 percent.

6.3 Personal Data Example

We run experiments for the real world example discussed earlier in the paper in Section 2. The schema $R$ is the same with 8 attributes and 8 privacy constraints. The workloads are generated at random as discussed in the previous subsection. Figure 9 depicts that for about 50 percent of the workloads, the percentage difference of the final result of hill
climbing from the initial partition is about 5 percent. Thus, given privacy constraints of the form listed for this example, it is easier to guess a reasonably valid and optimal decomposition for the schema. We also find that hill climbing terminates sooner (fewer number of iterations as compared to the synthetic data case) with close to 50 percent of the workloads terminating in a single iteration.

6.4 Varying Distributions of Privacy Constraints Generation Task

We had previously generated a synthetic dataset for the privacy constraints and the workload where all parameters were generated uniformly at random. For this experiment, we use a relation with 8 columns and 1000 tuples. There are 8 privacy constraints generated for the relation. There is a percentage weight parameter governing the generation of constraints which applies to the first three columns of the relation. For example, if percentage
weight is set to 20, then 60 percent \((20*3)\) of the time, one of the first three columns will be selected as a participant in the generated privacy constraint. So, setting this weight to around 12 results in a uniform distribution and as it goes above 33, we get a heavily biased set of privacy constraints on these three columns. We generate 30 workloads for each weight value that we desire to test. Figure 10 shows the average number of iterations as we vary the weight from 10 to 34 percent. It can be seen that as the bias on the three attributes increases, we get to the final result in fewer number of iterations. Fewer iterations are required because most of the privacy constraints are concerned with these three attributes and are independent of the others so we have fewer options to choose for these three attributes.

![Figure 10: Average Iterations with varying privacy constraints distribution](image)

7 Related Work

There is a wide consensus that privacy is a corporate responsibility [21]. In order to help and ensure corporations fulfill this responsibility, governments all over the world have passed multiple privacy acts and laws, for example, Gramm-Leach-Bliley (GLB) Act [13], Sarbanes-Oxley (SOX) Act [26], Health Insurance Portability and Accountability Act (HIPAA) [20], SB1386 [1] are some such well known U.S. privacy acts. Many use cases complying with these laws require organizations to encrypt the data in case it is hosted by an external service provider.

As discussed in the introduction, the outsourcing of data management has motivated the model where a DBMS provides reliable storage and efficient query execution, while not knowing the contents of the database [17]. Schemes proposed so far for this SaaS model [16] (Software as a Service) encrypt data on the client side and then store the encrypted database on the server side [16, 18, 6]. However, in order to achieve efficient query processing, all the above schemes only provide very weak notions of data privacy. In fact a server that is secure under formal cryptographic notions can be proved to be hopelessly inefficient for data processing [22].

After our original work [3] on using a combination of distribution and encryption for secure databases, our colleagues at Stanford University have developed partially [9] and
complete homomorphic encryption algorithms [12]. This allows arithmetic operations on ciphertext without decrypting to plaintext. Homomorphic encryption along with deterministic hashing primitives drastically improves SQL processing on encrypted data in the SaaS model.

[3, 23, 10] define privacy constraints and describe the architecture for secure database services. In this paper, we build on this architecture and develop algorithms to split the schema among the servers and perform distributed query processing for a subset of SQL (both are absent in [23]). K-anonymity [4] maintains aggregates while changing microdata for data publishing. This scheme maintains microdata intact for OLTP workloads also.

References


